

NOTES AND CORRESPONDENCE

A note on MOST and HOST for turbulence parametrization

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Email: bgrisog@gfz.hr**Abstract**

An extended expression of the (u_* , U) relationship (u_* is the square root of turbulent momentum fluxes and U is the mean wind speed) based on Monin–Obukhov Similarity Theory (MOST) is compared qualitatively with an observed (u_* , U) relationship for its variation between near neutral and stably stratified regimes. MOST, in its standard or classic form, cannot capture the observed characteristics of the increasing u_* with U from its gradual change in the statically (strongly) stable regime to the sharp change toward the near neutral regime, the so-called HOckey-Stick Transition (HOST). A heuristic formulation based on a drag coefficient is proposed for describing the observed relationship; it is found to be able to capture the tendency of the regime change even though the formulated transition is not as dramatic as the observations imply. For this purpose, almost like in some previous studies, a background flux is assumed. It accounts for cumulative non-traditional effects on turbulence; thus, it renders an extended MOST and HOST, arguably speaking, compatible.

KEYWORDS

drag coefficient, wind shear

1 | INTRODUCTION

Monin–Obukhov Similarity Theory (MOST) is a cornerstone of turbulence parametrization for modelling the atmospheric lower boundary in numerical weather prediction (NWP) and climate models, and its framework is used as guidance in observation analysis and interpretation. MOST has been widely investigated in the literature. The MO bulk formula for momentum performs reasonably well near the surface (e.g. Sorbjan and Grachev, 2010; Sorbjan, 2012; Sun *et al.*, 2016) and weak stratification conditions (Optis *et al.*, 2014). However, it is not adequate for observations away from the surface. In addition, its performance is questionable in extreme conditions, such as under very strong stratification (e.g. Grachev *et al.*, 2005; 2013), in the vicinity of a low-level jet (LLJ), or under weak

winds (Mahrt, 1998; 2008; 2014; Zilitinkevich and Calanca, 2000; Zilitinkevich and Esau, 2007; Sun *et al.*, 2012; 2016; Van de Wiel *et al.*, 2012) and for thin katabatic flows, which can be considered as a subset of LLJ (Grisogono and Oerlemans, 2001; Grisogono *et al.*, 2007; Oldroyd *et al.*, 2014). Nieuwstadt (1984) showed that local MOST may agree with observations better than classic MOST. Ansonge (2019) displays that classic MOST has no or even negative skill in locally estimating the stably stratified surface layer (SSL) properties from friction velocity, u_* , which is defined as $u_*(z) = [(u'w')^2 + (v'w')^2]^{1/4}$ (u' , v' , w' are turbulent fluctuations of the respective velocity components from $U(z)$ and the brackets (...) represent Reynolds averaging).

Various explanations regarding the performance of the MO bulk formula have focused on unsatisfactory conditions for validity of MOST such as non-stationarity (Klipp

and Mahrt, 2004; Grachev *et al.*, 2005; Mahrt, 2008; 2017), and gravity wave influences (Chimonas and Nappo, 1989; Steeneveld *et al.*, 2008). Some studies have gone further to investigate fundamental issues associated with MOST such as self-correlation (e.g. Klipp and Mahrt, 2004; Baas *et al.*, 2006; Ha *et al.*, 2007). So far, none of the explanations for the poor performance of MOST is conclusive.

Recently, Van de Wiel *et al.* (2012) and Sun *et al.* (2012) found the overall relationship between $u_*(z)$ and wind speed $U(z)$ resembles a hockey stick with the linear relationship as the stick, the upper part (strong $U(z)$), and the “stratified relationship” as the blade, the lower part (weak $U(z)$), Figure 1. Sun *et al.* (2016) named the transition between the stratified and the neutral relationship between u_* and $U(z)$ the HOckey-Stick Transition (HOST) and provided the HOST hypothesis to explain their observed turbulence characteristics under broad wind and stratification conditions. Relationships between u_* and $U(z)$ reflect dynamics in turbulence generation and the atmosphere stratification change in the surface layer (SL) between the surface and the observation height for $u_*(z)$ and $U(z)$. Under weak winds, the SL is strongly influenced by the surface heating or cooling, which is a component in the surface energy balance (SEB) as identified by Van de Wiel *et al.* (2012). When the SL is stratified, either stably or unstably, the only way to achieve the neutral SL is through mechanically generated turbulent mixing, which effectively transfers heat vertically. Practically, $U(z)$ changes are related to wind shear in its generation of mechanical turbulence; as a result, $U(z)$ is related to the SL stratification. Additional explanations for the observations are also provided (e.g. Van Hooijdonk *et al.*, 2015). Therefore, the HOST pattern has been observed over a variety of surface types at numerous sites. In our approach we implicitly assume an equilibrium state of flow.

Sun *et al.* (2016) demonstrated that under statically near neutral conditions, $u_*(z)$ is approximately linearly related to $U(z)$ as (their eq. (12)):

$$u_*(z) = \alpha(z)U(z) + \beta(z), \quad U(z) > V_s(z), \quad (1)$$

which is illustrated by the thick lines in Figure 1. In Equation 1, $\alpha(z)$ represents the square root of the drag coefficient, and $\beta(z)$ is the intercept of the linear neutral relationship between $u_*(z)$ and $U(z)$. They observed that $\alpha(z)$ varies only with z below 10 m and remains nearly invariant for $z > 10$ m, while $\beta(z)$ is mostly negative and monotonically approaches zero as $z \rightarrow z_0$. As also explained by Sun *et al.* (2016), the near neutral regime can be achieved from the SSL only through vertical redistributions of the cold air from the radiatively cooled surface by mechanically generated turbulence. To remove the stable stratification between the surface and z , $U(z)$ needs to exceed a required

minimum of $U(z)$, $V_s(z)$, such that the shear-generated turbulent mixing is sufficiently strong within the layer below z . Thus, $\beta(z)$ varies with z : $\beta(z_0) = 0$, $\beta(z > 0) < 0$ in the SSL and slightly $\beta(z > 0) > 0$ under unstable stratification. (see fig. 13c in Sun *et al.*, 2016); the roughness length, z_0 , is used here interchangeably with $z = 0$.

Sun *et al.* (2016) also found that $\alpha(z)$ only varies very near the surface and remains nearly invariant with z . Physically, the surface drag coefficient by definition, that is, $C_d \equiv (u_*(z)/U(z))^2$, represents u_* variations with U under strong winds associated with statically near neutral conditions when turbulent mixing at z is fully coupled with the surface.

In this research note, we illustrate that the observed HOST pattern could be heuristically obtained by keeping the important physical understanding of the observed HOST pattern, that is, (a) $U(z)$ and $u_*(z)$ variations are related to the SL stratification, and (b) the approximately invariant α is associated with C_d . Hence, Equation 1 is only our starting point. Below we make a single step toward relating MOST to HOST for better agreement with observations so that any researcher can choose on his/her own how to interpret the proposed extended bulk formula.

2 | NONLINEAR APPROACH AND MINIMUM OR BACKGROUND FLUX

To keep C_d (at least nearly) invariant and in accordance with MOST, we start with a simple quadratic form for u_* (e.g. Stull, 1988; Kundu and Cohen, 2002), with an additional term (for simplicity and compactness, we do not write dependency on z for the first terms on the following both sides), that is,

$$u_*^2 = C_d U^2 + u_{*0}^2 \quad (2)$$

at a given z , where C_d is roughly a constant for a given surface and u_{*0} plays an important role in the SL at low $U(z)$; namely, u_{*0}^2 is a minimum or background momentum flux accounting for unresolved sub-grid, mostly non-turbulent, motions (e.g. short buoyancy waves, semi-chaotic thin drainage flows, unsteadiness, etc.). As such, u_{*0}^2 is a cumulative quantity independent of z ; it also represents the kinetic energy required to reach the neutral (u_* , U) relationship and it is not u_* at z_0 . One might call Equation 2 plausibly an extended or slightly modified version of MOST. In comparison with Equation 1, we have

$$C_d U^2 + u_{*0}^2 = \alpha^2 U^2 + 2\alpha\beta U + \beta^2 \quad (3)$$

for $U > V_s$. That is, $\alpha(z) = C_d^{1/2}$ (e.g. Mahrt *et al.*, 2015), then $\beta(U, u_{*0}, C_d) = -C_d^{-1/2} U \pm [C_d U^2 + u_{*0}^2]^{1/2}$. To

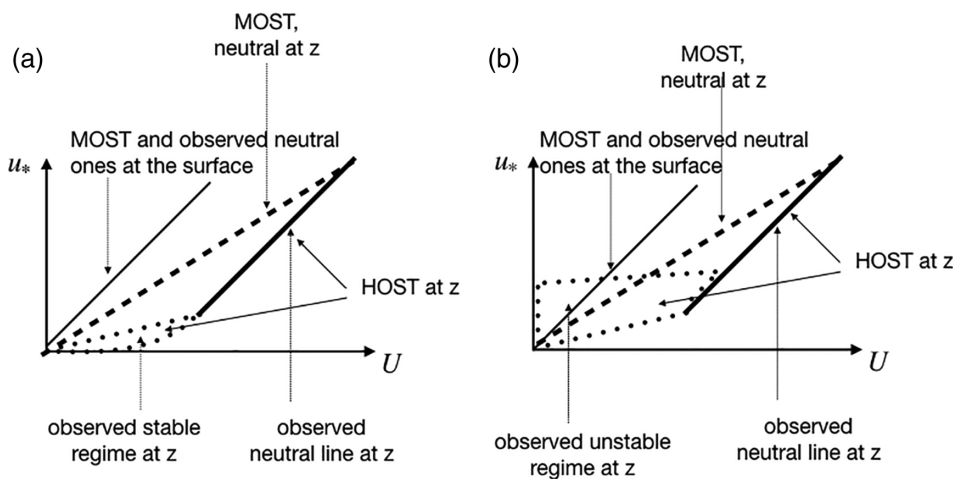


FIGURE 1 Schematic of the observed (u_* , U) relationship at a given observation height z for typical (a) night-time and (b) daytime conditions. The observed neutral (u_* , U) lines at z are marked with the thick solid lines, and the observed stratified regimes are shaded grey in both (a) and (b). The slope and the intercept of the observed neutral (u_* , U) lines are marked as $\alpha(z)$ and $\beta(z)$, respectively. The observed transition between the stratified (either stable or unstable) and the neutral regimes at z represent the observed HOST. Thick dashed line represents the neutral (u_* , U) line from the MO bulk formula

explore u_* in the stratified and neutral regimes, we consider Equation 2 regarding (a) $U > V_s$ and (b) $U < V_s$:

$$u_*^2 = C_{1d}U^2 + u_{*0}^2, \quad U > V_s, \quad (4a)$$

$$u_*^2 = C_{2d}U^2 + u_{*0}^2, \quad U < V_s. \quad (4b)$$

It is assumed that $u_{*0} < C_{1d}^{1/2}U$; moreover, since C_{1d} and C_{2d} can be the same, the approach is still valid because it is intrinsically based on U vs. V_s but not on a plausible small difference between C_d values. It makes sense now to assess Equation 2 with respect to $\min(U)$ value for sustainable turbulence, V_s ; C_{1d} and C_{2d} are drag coefficients; according to Mahrt *et al.* (2015) typically but not necessarily $C_{1d} \geq C_{2d}$. The distinction between C_{1d} and C_{2d} (say, for near neutral and stratified regime) kept here for generality purpose only, seems to be less apparent for lower measurement heights than for higher ones, because the higher measurement levels have larger V_s , see for example fig. 1 in Sun *et al.* (2012), so there the flow remains longer in the regime described by Equation 4b. The crude matching condition at a critical u_* is $u_{*c}^2 = C_{dd}V_s^2 + u_{*0}^2$, with C_{dd} as a suitable mean value of C_d for the two regimes; most simply, $C_{dd} = C_d$. Consequences of this simplified matching and the meaning of V_s are described in Section 2.4. Next, we focus on the asymptotic solutions of Equation 2 under the near neutral regime associated with strong wind $U > V_s$ and under the stratified regime associated with weak winds $U < V_s$. Once again, plausible but definitely only small potential changes of C_d , if any, are not an important point here. We keep it so for generality as one or two, that is, the same or very close, values of the parameter C_d .

2.1 | Higher wind speeds, $U > V_s$

The solution to Equation 2 toward Equation 4a, where the first term on the RHS is larger than the second term and $U > V_s$, reads approximately as

$$u_* = C_{1d}^{1/2}U + \frac{u_{*0}^2}{2C_{1d}^{1/2}U} + H.O.T. \quad (5)$$

where *H.O.T.* means higher order terms (the first *H.O.T.* is $-u_{*0}^4/[8C_{1d}^{3/2}U^3]$). Note that Equation 5 is a valid asymptotic solution to Equations 2 and 4a and there is a linear relationship between u_* and U for sufficiently large U ; moreover, $\alpha = C_{1d}^{1/2}$ as needed. The variation of the equivalent β term depends on the SL stratification, which is observed in Mahrt *et al.* (2013).

2.2 | Lower wind speeds, $U < V_s$

Very stable flow regime can be defined as winds weaker than the threshold V_s (Sun *et al.*, 2012; 2016; Van de Wiel *et al.*, 2012; Mahrt *et al.*, 2013). Turbulence intermittency might appear due to for example, external motions, the mean flow switches across $U > V_s$ and $U < V_s$, etc. The solution to Equations 2 and 4b where the second term on the RHS is larger than the first term and $U < V_s$ reads approximately as

$$u_* = u_{*0} + \frac{C_{2d}U^2}{2u_{*0}} - H.O.T. \quad (6)$$

Now Equation 6 is a valid asymptotic solution to Equations 2 and 4b for sufficiently small U where there

is a weak parabolic (quadratic) relationship in Equation 6 between u_* and U . It actually means a very weak u_* dependence on U in this sub-regime where U is relatively small. This is the key point. As Equation 6 suggests, u_* remains almost a constant nearly equal to u_{*0} , under a very low U regime, which agrees with Mauritsen *et al.* (2007), Van de Wiel *et al.* (2012), Mahrt *et al.* (2015), Sun *et al.* (2016), Mahrt (2017), etc. Thus Equations 2, 5 and 6 assure a ski-like behaviour of $u_*(U)$, that is, an analytic, that is, a smoothed hockey-stick form, in a simple and appealing way. The above analyses demonstrate that to keep $C_d \approx \text{constant}$ (choose near-neutral value, C_{1d}), u_* has to vary nonlinearly with $U(z)$. The simple expression of Equation 2 can capture this important feature in the observations mentioned in the Introduction.

2.3 | Simple matching of speeds

The simple matching condition, or more precisely here, patching (Bender and Orszag, 1978; Grisogono and Oerlemans, 2001) given below Equation 4b, suggests that both contributions to u_{*c} are of the same order at $U = V_s$; otherwise, $V_s(z)$ would not be such a significant variable. Above $V_s(z)$ it is $U(z)$ that dominates the behaviour of u_* . Thus, Equation 2, through its asymptotic solutions Equations 5 and 6, qualitatively resembles HOST because it possesses a ski-like, or a smooth hockey stick-like behaviour (without a sharp kink). Basically, Equations 5 and 6 are patched at $V_s(z)$ via their consecutive limits: the former as its lower limit, and the latter as its higher limit, respectively. As shown in Figure 2, the ski-like pattern is obtained using Equation 2; notice that a single constant C_d is used since the approach deployed does not depend crucially on an exact value of C_d . Almost needless to say, Figure 2, that is, its ski-like pattern, agrees with the related findings of Mahrt *et al.* (2015, their figs. 4, 5a and 8b) and some other studies already stated.

2.4 | Estimation of V_s

From Van de Wiel *et al.* (2012) using the KNMI Cabauw observatory data, their figure 1, V_s at 40 m for minimum u_*^2 at 5 m can be approximated to be between 1 and about 6 $\text{m}\cdot\text{s}^{-1}$. This broadly agrees with Van Hooijdonk *et al.* (2015). Likewise, Sun *et al.* (2012), their figure 1 using the CASES-99 dataset, imply that V_s can be between ~ 1 and 8.5 $\text{m}\cdot\text{s}^{-1}$ for the elevations between 0.5 and 55 m. More firmly, Sun *et al.* (2016) from the same dataset in their Figure 4, red vertical lines, claim night-time V_s to be $\sim 3.5 \text{ m}\cdot\text{s}^{-1}$ at 5 m and $\sim 4.8 \text{ m}\cdot\text{s}^{-1}$ at 10 m height; the corresponding u_{*c} are estimated here as 0.2 and 0.25 $\text{m}\cdot\text{s}^{-1}$,

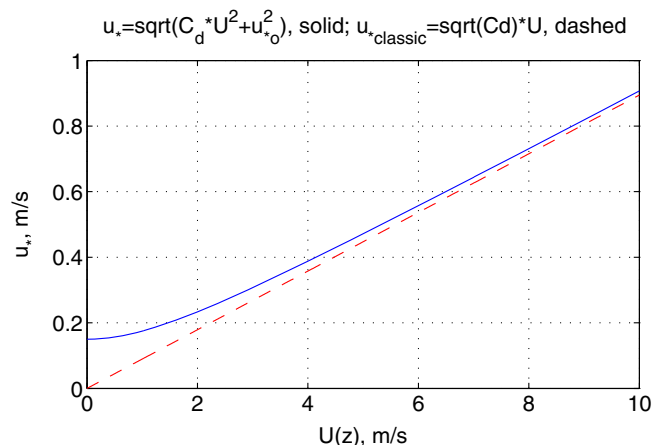


FIGURE 2 Qualitative resemblance of HOST behaviour: ski-like dependency $u_*(U(z))$ based on Equation 2, solid, similar to a smoothed hockey-stick, to Mahrt *et al.* (2015, their figs. 4, 5a and 8b), Van de Wiel *et al.* (2012, their figs. 1 and 4, though they used u_*^2 for the vertical axis) and Sun *et al.* (2016, their figs. 4a and 13a). This is a qualitative definition of HOST showing a smooth transition between two regimes, at weak and strong wind speeds (near-neutral stratification), respectively. From Equation 7 and discussion below it follows here $(C_d)^{1/2} V_s \approx u_{*0}$; thus, $V_s \approx 1.68 \text{ m}\cdot\text{s}^{-1}$. Classical linear relation based on MO bulk formula is in dashed line (its stability functions may do only minor to moderate corrections). The values used are $C_d = 0.008$ and $u_{*0} = 0.15 \text{ m}\cdot\text{s}^{-1}$ [Colour figure can be viewed at wileyonlinelibrary.com]

respectively. But a closer look at their figure 4 suggests that those V_s values should be somewhat smaller. The other particular V_s values at nine levels are marked in their figure 5, and from there we more firmly re-estimate V_s as $\approx (3 \pm 0.2) \text{ m}\cdot\text{s}^{-1}$ at 5 m and $\approx (4.3 \pm 0.2) \text{ m}\cdot\text{s}^{-1}$ at 10 m height; thus, the correspondingly re-estimated u_{*c} are $\approx (0.14 \pm 0.01) \text{ m}\cdot\text{s}^{-1}$ and $(0.18 \pm 0.01) \text{ m}\cdot\text{s}^{-1}$, respectively.

Once again, the patching condition between Equations 4a and 4b is

$$u_{*c}^2 = C_{dd} V_s^2 + u_{*0}^2 \quad \text{for } U = V_s, \quad (7)$$

where all the symbols are already defined; at this critical value of u_* , that is, u_{*c} , it should be expected that both terms on the right-hand side (RHS) of Equation 7 contribute equally to u_{*c} . Consequently, $u_{*c} = (2C_{dd})^{1/2} V_s = 2^{1/2} u_{*0}$. The latter may be used in the asymptotic forms of u_* , Equations 5 and 6, with various combinations of controlling variables u_{*c} , V_s and u_{*0} , yielding further insights into their meanings.

Three simple examples follow. (a) Assume $C_{dd} = 1.6 \cdot 10^{-3}$ and $V_s = 2 \text{ m}\cdot\text{s}^{-1}$; then $u_{*0} = (C_{dd})^{1/2} V_s = 8 \text{ cm}\cdot\text{s}^{-1}$ and $u_{*c} \approx 11.3 \text{ cm}\cdot\text{s}^{-1}$. Beyond the latter value u_* behaves in the linear fashion with respect to $U(z)$ in accordance with Equations 4a and 5. On the contrary, in the range $u_{*0} \leq u_* \leq u_{*c}$, that is, in our example, 8

$\text{cm}\cdot\text{s}^{-1} \leq u_* \leq 11.3 \text{ cm}\cdot\text{s}^{-1}$, u_* obeys Equations 4b and 6 which relates to strongly stratified turbulence regime. It is an interesting and most likely accidental coincidence that in the Meteorologiska Institution, Uppsala Universitet (MIUU) mesoscale model (e.g. Enger and Grisogono, 1998; Grisogono and Enger, 2004) the minimum u_* was set to $7 \text{ cm}\cdot\text{s}^{-1}$. (b) Using the estimated pairs of (V_s, u_{*c}) from Sun *et al.* (2016), their figures 4 and 5, we find C_{dd} values at 5 and 10 m to be $\approx 1.1 \times 10^{-3}$ and 1×10^{-3} , respectively. Moreover, the corresponding u_{*0} at 5 and 10 m is now calculated to be $\approx (10 \pm 1)$ and $(13 \pm 1) \text{ cm}\cdot\text{s}^{-1}$, respectively. (c) From two different night-time datasets with significant directional shears, figure 1b in Mahrt (2017), we estimate V_s and u_{*c} at 5 m to be $\approx (1.25 \pm 0.1) \text{ m}\cdot\text{s}^{-1}$ and $\approx (7 \pm 1.5) \text{ cm}\cdot\text{s}^{-1}$, respectively; then we find analytically $u_{*0} \approx (5 \pm 1) \text{ cm}\cdot\text{s}^{-1}$, which agrees with the estimation from fig. 1b in Mahrt (2017).

Very weak turbulence in the SSL occurs with large Ri_b (Mahrt, 1998; Grachev *et al.*, 2005; 2013; Mauritsen *et al.*, 2007; Van de Wiel *et al.*, 2012). Considering the meaning of V_s , one may define the Ri_b threshold value for the SSL

$$Ri_{b0} = \left(\frac{g\Delta\theta\Delta z}{\Theta} \right) / V_s^2 \quad (8a)$$

where $\Delta\theta$ is potential temperature difference between the upper and lower temperature level separated by distance Δz ; this is very similar to Mahrt *et al.* (2013), their equation (1); 2015). It follows after Equation 7 that

$$Ri_{b0} = \left(\frac{2C_{dd}g\Delta\theta\Delta z}{\Theta} \right) / u_{*c}^2. \quad (8b)$$

For $Ri_b < Ri_{b0}$, relatively strong stratified turbulence is expected and vice versa for $Ri_b > Ri_{b0}$. If we assume Ri_{b0} around unity, that is, $Ri_{b0} = (1 \pm \delta)$, $|\delta| < 1$, then Equation 8 suggests the critical value u_{*c} for continuously sustained turbulence below which only more sporadic, weak yet strongly stratified turbulence occurs:

$$u_{*c} \approx \left(\frac{2C_{dd}g\Delta\theta\Delta z}{\Theta} \right)^{1/2} \left(1 \mp \frac{\delta}{2} \right) \quad (8c)$$

for the parameters given within the layer Δz . Here δ loosely emulates the variation of Ri_{b0} due to a multitude of sub-mesoscale motions affecting and eventually inducing strongly stratified turbulence; the larger Ri_{b0} implies the smaller u_{*c} , that further complicates parametrizations of strongly stratified turbulence in numerical models.

3 | CONCLUSION

Some basic properties of the stably stratified surface layer, SSL, are addressed. We discuss certain observations shown

by Van de Wiel *et al.* (2012), Mahrt *et al.* (2015), Van Hooijdonk *et al.* (2015), Sun *et al.* (2012; 2016) and Mahrt (2017). Our analytical reasoning, using only the mean wind speed, suggests that at first sight HOST and MOST may seem incompatible (SEB effects excluded). However, when certain points on both sides are relaxed, a qualitative agreement between the views on the SSL might ensue. In other words, slightly modified MOST allowing for a background or at least somehow finding a minimum u_* , that is, u_{*0} , that is due to a multitude of (sub-)mesoscale features including unsteadiness, background stratification, short (often models' unresolved) waves, heat effects, etc., may emulate the characteristic of HOST where u_* depends on $U(z)$ in a ski-like or smooth hockey-stick pattern as presented in numerous previous studies. There, u_{*0} accounts for cumulative non-traditional effects on turbulence, all lumped together, that is missing in the standard MOST approach.

Two sub-regimes are identified here analytically: for very low $U(z)$, u_* is small, yet finite and almost $U(z)$ -independent, but for moderate and large $U(z)$, u_* grows linearly with $U(z)$. Hence, one of the first analytic steps, relating HOST to the extended MOST, is obtained in terms of u_* . It should be kept in mind that Sun *et al.* (2016)'s HOST hypothesis and MOST are different by definition; the contradictions are still not fully solved, which requires more work in the future. Nevertheless, the progress is that the proposed consideration of the minimum or background friction velocity provides a more flexible mathematical framework for suitable interpretations and applications. Perhaps both the HOST hypothesis and MOST can be combined with the concept of background u_* , that is, u_{*0} , on their own; this would lead to better agreement with observations and aid turbulence parametrization.

A new result is that a slightly modified drag coefficient method, that is generally compatible with MOST, produces a ski-like (i.e. smooth hockey-stick) behaviour at least in a qualitative sense, which is an essential HOST feature. If Equation 1 is sort of, or very loosely, generalized as Equations 2, 5 and 6, one just might find MOST as a subset, or a special case of HOST in future research (or vice versa if MOST will be more extended). In the flux form, the threshold $U(z)$ for sustainable continuous turbulence in the SSL ensues asymptotically as a function of drag coefficient and a minimum (or more broadly, background), or alternatively, critical friction velocity. The result can be important for turbulence data analysis and numerical models that typically exclude some of the smallest mesoscale processes which may contribute to turbulence in non-traditional ways.

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